ME 314 - Engineering Design : Mechanical Components
Lecture 15

Note Title

An important question is whether or not the endurance limit, $S_{e'}$ is correlated with the ultimate tensile strength, S_{ut}. Here are some approximate relations:

Steels:	$S_{e'} = 0.5\ S_{ut}$	for	$S_{ut} < 200$ kpsi (1400 MPa)
	$S_{e'} = 100$ kpsi (700 MPa)	for	$S_{ut} \geq 200$ kpsi (1400 MPa)
Irons:	$S_{e'} = 0.4\ S_{ut}$	for	$S_{ut} < 60$ kpsi (400 MPa)
	$S_{e'} = 24$ kpsi (160 MPa)	for	$S_{ut} \geq 60$ kpsi (400 MPa)
Aluminum:	$S_{f'\ at\ 5x10^8} = 0.4\ S_{ut}$	for	$S_{ut} < 48$ kpsi (330 MPa)
	$S_{f'\ at\ 5x10^8} = 19$ kpsi (130 MPa)	for	$S_{ut} \geq 48$ kpsi (330 MPa)
Copper alloys:	$S_{f'\ at\ 5x10^8} = 0.4\ S_{ut}$	for	$S_{ut} < 40$ kpsi (280 MPa)
	$S_{f'\ at\ 5x10^8} = 14$ kpsi (100 MPa)	for	$S_{ut} \geq 40$ kpsi (280 MPa)

Correction Factors for $S_{e'}$ and $S_{f'}$ Data

Laboratory tests on the rotating beam specimens are conducted very carefully under controlled conditions. For this reason, the endurance limit of a machine part is generally different from the value obtained in the lab. Thus, some corrections due to material (chemical composition, etc.), manufacturing (surface condition, stress concentration, etc. environment (temperature, etc.), and design (shape, size, etc.) are necessary. We write,

$$S_e = C_{load}\ C_{size}\ C_{surf}\ C_{temp}\ C_{reliab}\ S_{e'}$$

$$S_f = C_{load}\ C_{size}\ C_{surf}\ C_{temp}\ C_{reliab}\ S_{f'} \qquad (6.6)$$

where S_e is the corrected endurance limit for a material that has one and

S_f is the corrected fatigue strength at a particular cycle (e.g. $5x10^8$) for a material (e.g., aluminum) that does not have an endurance limit.

C's are strength reduction (or correction) factors and are defined as follows:

Load Factor, C_{load}

bending: $C_{load} = 1$

axial loading: $C_{load} = 0.7$ (6.7a)

Size Factor, C_{size}

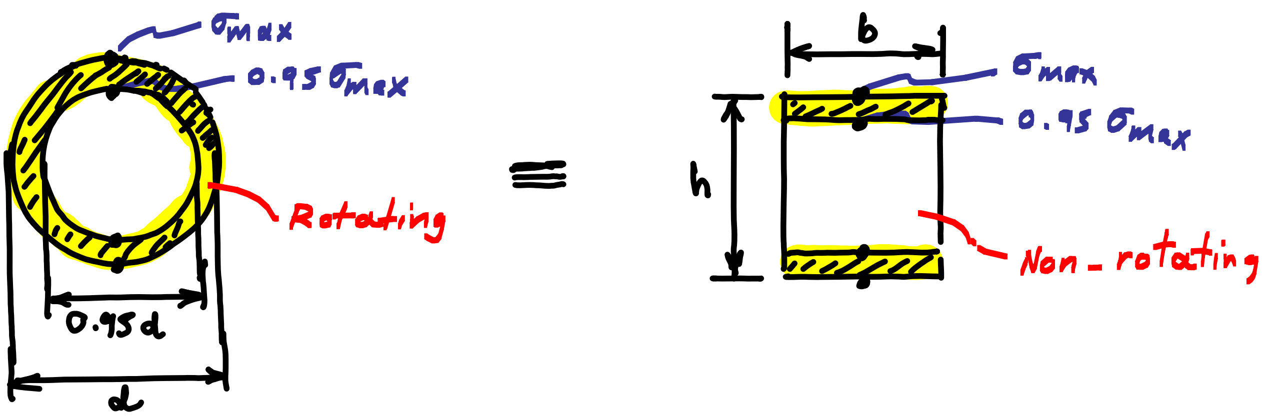
If the component has a size different from the rotating-beam specimen (i.e., 0.3-in diameter) we must make a correction. For cylindrical steel parts, Shigley & Mischke propose:

For	$d \leq 0.3$ in (8 mm)	$C_{size} = 1$
For	$0.3 < d < 10$ in	$C_{size} = 0.869\, d^{-0.097}$
		(6.7b)
For	8 mm $< d <$ 250 mm	$C_{size} = 1.189\, d^{-0.097}$
For larger sizes		$C_{size} = 0.6$

For non-rotating parts, or parts with non-circular cross-section, an "equivalent diameter" is defined and is used in the above equations. as follows:

[Cross-sectional area stressed above 0.95 σ_{max}] non-round / non-rotating part =

[Cross-sectional area stressed above 0.95 σ_{max}] rotating-beam specimen



$$A_{95} = \frac{\pi}{4}\left[d^2 - (0.95d)^2\right] = 0.0766\, d^2$$

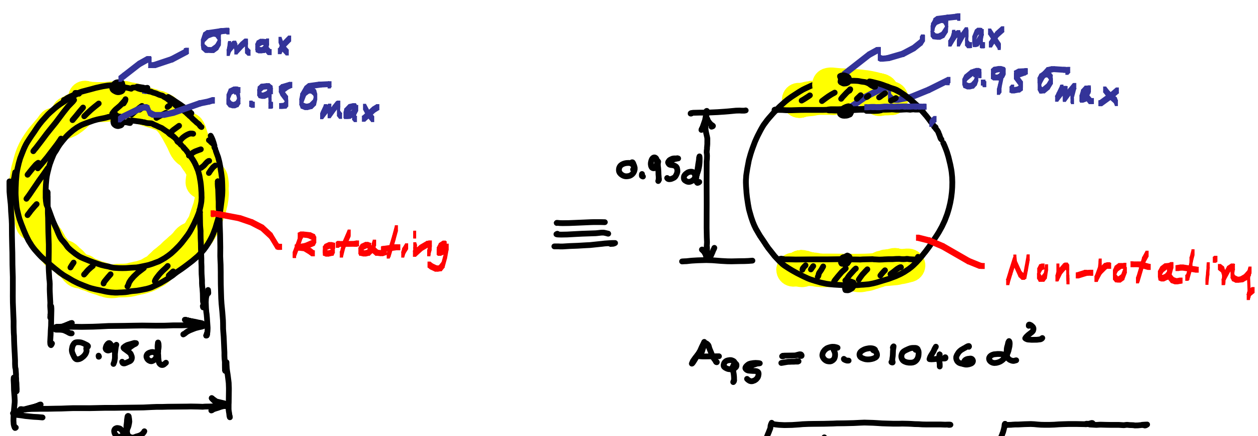
$$A_{95} = 0.05\, bh$$

$$\therefore \quad d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}} \qquad (6.7d)$$

For a rectangular cross-section (non-rotating):

$$d_{equiv} = \sqrt{\frac{0.05\,bh}{0.0766}}$$

For a non-rotating round section we have:



$$A_{95} = 0.0766\,d^2$$

$$A_{95} = 0.01046\,d^2$$

$$\therefore\; d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.01046}{0.0766}} = 0.370\,d$$

(d_{equiv}: Effective size of a non-rotating round beam)

Surface Factor, C_{surf}

To avoid stress raisers, surface of the rotating-beam specimen is polished to a mirror finish. A real part, however, does not have such an expensive finish. There are various ways to correct for the surface finish. Here, we use the equation proposed by Shiegly and Mischke:

$$C_{surf} = A(S_{ut})^b, \qquad \text{if } C_{surf} > 1, \quad \text{set } C_{surf} = 1 \qquad (6.7e)$$

The coefficient *A* and exponent *b* for various finishes are given in Table 6-3.

Surface Finish	MPa A	MPa b	kpsi A	kpsi b
Ground	1.58	–0.085	1.34	–0.085
Machined or cold-rolled	4.51	–0.265	2.7	–0.265
Hot-rolled	57.7	–0.718	14.4	–0.718
As-forged	272	–0.995	39.9	–0.995

Table 6-3

Coefficients for the Surface-Factor Equation Source: Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989, p. 283 with permission.

See text for more details on other surface properties.

Temperature Factor, C_{temp}

The rotating-beam test is done at room temperature. At temperatures above 50% of the melting point, creep becomes a significant factor & strain-life (Epsilon-N) approach should be used instead of stress-life (S-N) approach. For moderately high temperatures, Shiegley and Mitchell suggest the following:

For $T \leq 450^oC$ (or 840^oF), $C_{temp} = 1$ (6.7f)

For $450^oC \leq T \leq 550^oC$, $C_{temp} = 1 - 0.0058 (T-450)$

For $840^oF \leq T \leq 1020^oF$, $C_{temp} = 1 - 0.0032 (T-840)$

These are for steel only.

Reliability Factor, C_{reliab}

The strength data reported in tables are mean values. Thus for a 50% reliability Creliab = 1.

This factor reduces for a higher reliability according to Table 6-4.

See text for other factors such as the environment.

Reliability %	C_{reliab}
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

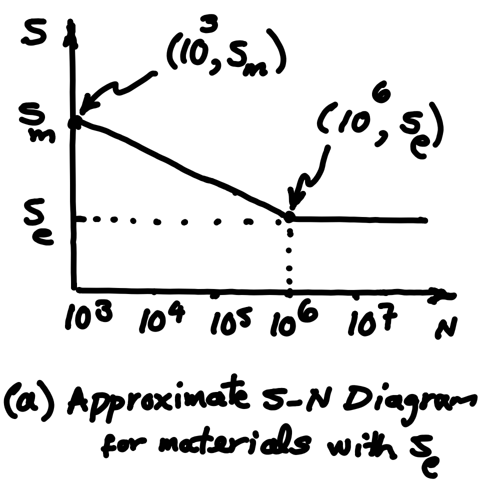
Table 6-4

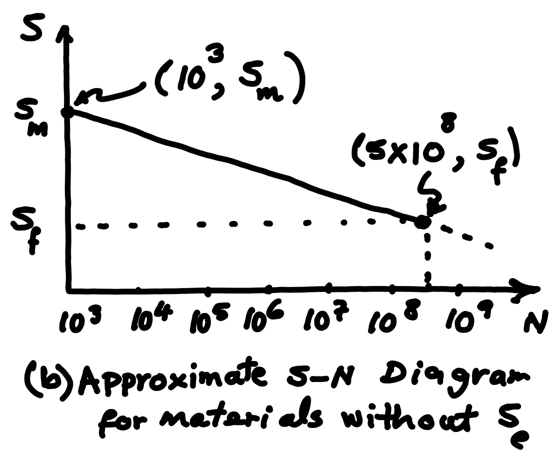
Reliability Factors for $S_d = 0.08\ \mu$.

Creating Approximate S-N Diagrams

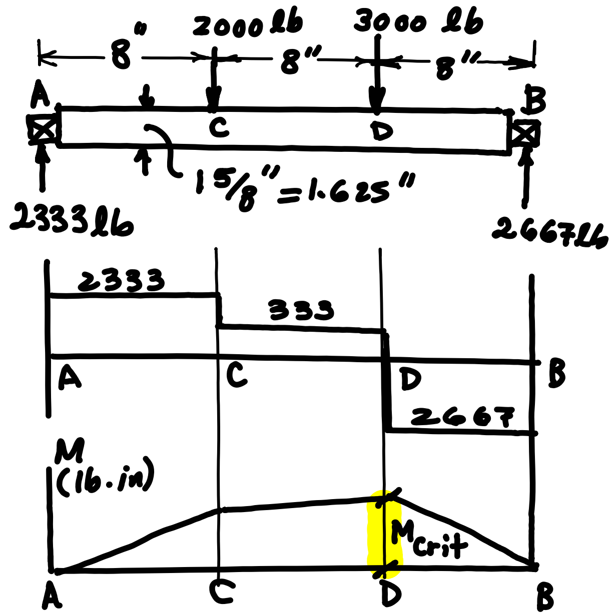
The S-N diagram, as mentioned earlier in this lecture, consists of a HCF region (N > 1000) and a LCF (1< N < 1000) region. For HCF, we have Eq. (6.6) for the endurance limit S_e, or the fatigue strength, S_f. Let the material strength at 1000 cycles be called S_m. Test data indicate that the following estimates of S_m are reasonable:

An approximate (S-N) diagram is obtained by connecting S_m given by (6.9) and the corrected endurance limit S_e from (6.6). If the material does not have an endurance limit, S_m is connected to the corrected fatigue strength, S_f, given by (6.6) as shown in Fig. (b) on the right. The equation of the line from S_m to S_e or S_f can be written as (recalling that this is a log-log plot):





Example: **The steel shaft shown has a ground finish and a tensile strength of 89 kpsi. The shaft rotates at 1720 rpm and is supported in rolling bearings at *A* and *B*. Estimate the life of the part under the loads shown with 99.99% reliability.**



1) Stress at the critical section:

2) Endurance Limit:

Load Factor: Eq. 6.7a

Size Factor: Eq. 6.7b

Surface Factor: Eq. 6.7e

Temperature Factor: Eq. 6.7f

Reliability Factor: Table 6-4

3) Life :

6-7 Notches and Stress Concentration

Notch **is used here as a generic term for a stress raiser. It can be a hole, a groove, a fillet, an abrupt change in cross-section, etc. In the case of static loading, notches are of concern only for brittle materials. With dynamic loads, stress concentration is important for both ductile and brittle materials to a various degree. In general, the more ductile the material, the less notch sensitive, it is. Brittle materials are more notch sensitive. The notch sensitivity q is defined as**

$$q = \frac{K_f - 1}{K_t - 1} \qquad (6.11a)$$

where K_t **= Theoretical (Static) Stress-Concentration Factor**

and

K_f **= Fatigue (Dynamic) Stress-Concentration Factor**

The notch sensitivity q varies between 0 and 1 and its value can be obtained from the formula:

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \qquad (6.13)$$

where

$\sqrt{a}$ = Neuber's Constant given in Table 6-6

r = Notch radius

For torsion use (S_{ut} + 20 kpsi)

S_{ut} (ksi)	$\sqrt{a}$ (in$^{0.5}$)
50	0.130
55	0.118
60	0.108
70	0.093
80	0.080
90	0.070
100	0.062
110	0.055
120	0.049
130	0.044
140	0.039
160	0.031
180	0.024
200	0.018
220	0.013
240	0.009

Table 6-6
Neuber's Constant for Steels.

The procedure is to first obtain K_t using the graphs in Appendix E (pp 955-962), then find q from (6.13) and use them in equation

$$K_f = 1 + q\,(K_t - 1) \qquad (6.11b)$$

to find the fatigue S-C factor.

The dynamic stress is then increased by the factor K_f for tensile stress and K_{fs} for shear stress.

$$\sigma = K_f\,\sigma_{nom} \quad , \quad \tau = K_{fs}\,\tau_{nom} \qquad (6.12)$$

The notch sensitivity q given by (6.13) is plotted for steel and aluminum in Fig. 6-36.

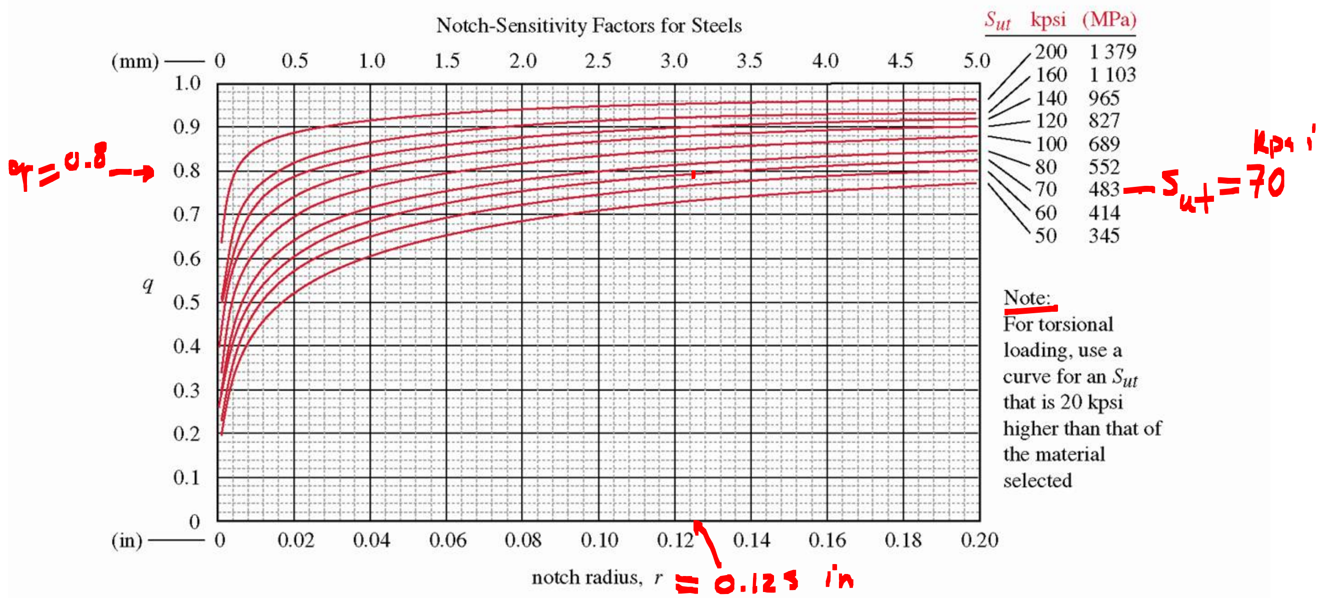


Figure 6-36 Part 1

Notch-Sensitivity Curves for Steels Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in *Metal Fatigue* by G. *Sines* and J. Waisman, McGraw-Hill, New York, 1959.

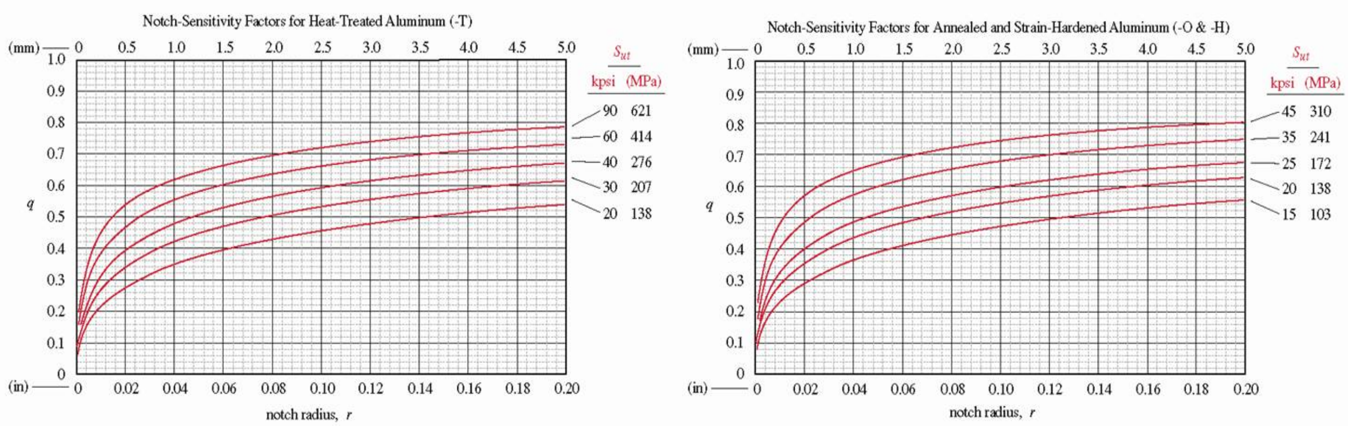
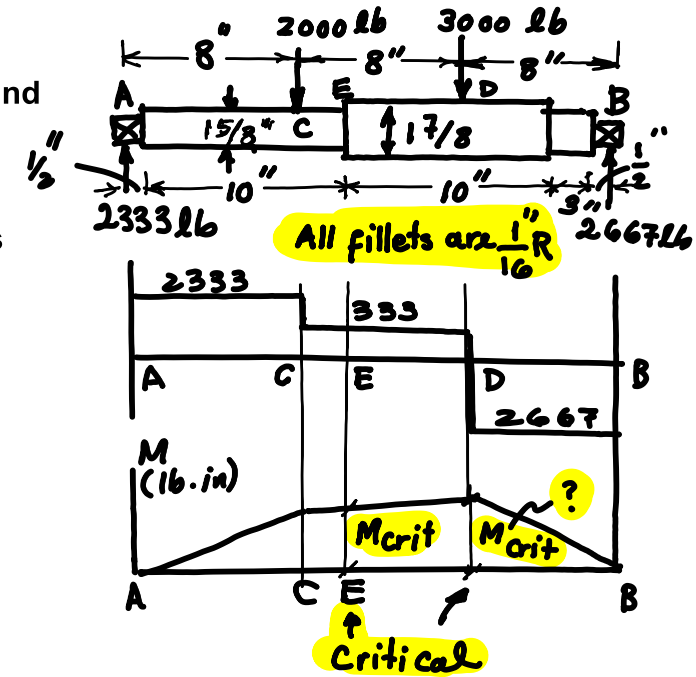


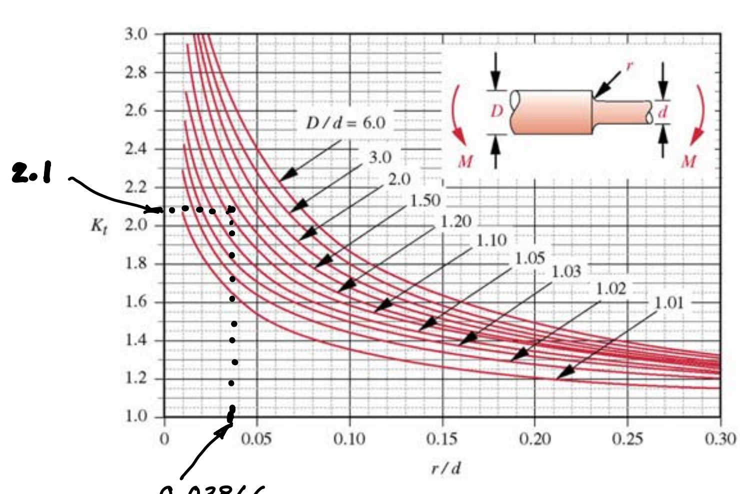
Figure 6-36 Part 2

Notch-Sensitivity Curves for Aluminums Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in *Metal Fatigue* by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

Example: **The steel shaft shown has a ground finish and a tensile strength of 89 kpsi. The shaft rotates at 1720 rpm and is supported in rolling bearings at A and B. Estimate the life of the part under the loads shown with 99.99% reliability.**



1) Stress at the critical section



$K_t \equiv A\left(\frac{r}{d}\right)^b$

where :

D/d	A	b
6.00	0.878 68	−0.332 43
3.00	0.893 34	−0.308 60
2.00	0.908 79	−0.285 98
1.50	0.938 36	−0.257 59
1.20	0.970 98	−0.217 96
1.10	0.951 20	−0.237 57
1.07	0.975 27	−0.209 58
1.05	0.981 37	−0.196 53
1.03	0.980 61	−0.183 81
1.02	0.960 48	−0.177 11
1.01	0.919 38	−0.170 32

Figure E-2

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Bending.

2) Endurance Limit :

$$S_e = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_e'$$

3) Life :